# 20-15 LATERAL SPREADING ANALYSIS FOR NEW AND EXISTING BRIDGES

## Calculation of Foundation Loads Due to the Soil Crust

Loads on the foundation due to the down slope movement of the soil crust often dominate other loads. The interaction between the foundation and soil crust can be modeled using user-specified *p-y* curves in a pile lateral load analysis program or FEM software. A trilinear force-deflection model, shown in Figure A1, is recommended as the basis for the *p-y* curves. This model is defined by the parameters $F_{ULT}$ and $\Delta_{max}$ which represent the ultimate crust load on the pile cap or composite cap-soil-pile block (see discussion below in Determination of Critical Failure Surface) and the relative soil displacement required to achieve $F_{ULT}$, respectively. The determination of these parameters is described below. Once the force-displacement relationship is established, a *p-y* curve can be defined by dividing the force by the pile cap or composite block thickness.
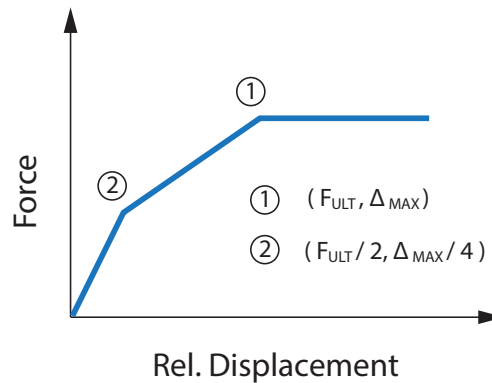


**Figure A1  Idealized force-deflection behavior of the pile cap. The trilinear curve is defined by the parameters $F_{ULT}$ and $\Delta_{MAX}$**

## Definition of Dimension Parameters

A typical foundation configuration and reference dimensions are provided in Figure A2. $W_L$ and $W_T$ refer to the longitudinal and transverse pile cap widths, respectively. *D* is the depth from ground surface to the top of pile cap and *T* is the pile cap thickness.
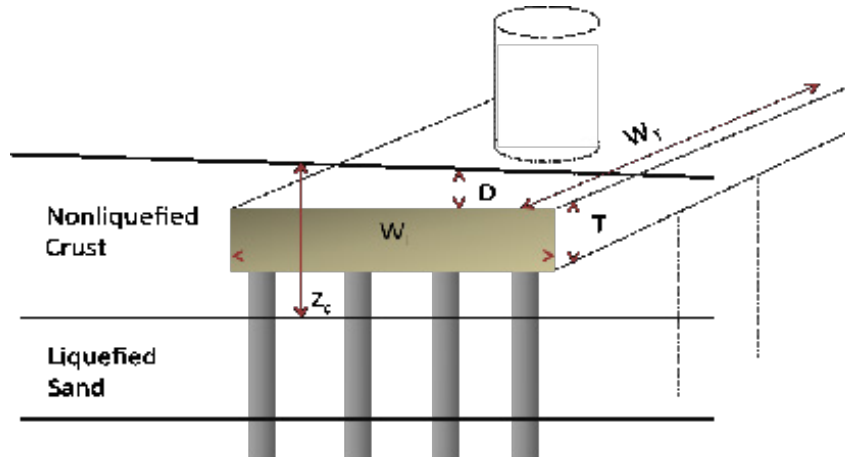
**Figure A2  Pile foundation schematic with transverse and longitudinal width dimensions $W_T$ and $W_L$, pile cap thickness $T$, depth to top of cap $D$, and crust thickness $Z_C$**

## Determination of $F_{ULT}$

The maximum crust load on the pile cap can be calculated according to equation (1):

$$F_{ULT} = F_{PASSIVE} + F_{SIDES} \tag{1}$$

In this equation, $F_{PASSIVE}$ refers to the passive force resulting from the compression of soil on the up slope face of the foundation and $F_{SIDES}$ refers to the friction or adhesion of the soil moving along the side of the foundation.  Note that a friction force below the pile cap caused by soil flowing through the piles is ignored along with a possible active force on the down slope side of the foundation (acting up slope).  These forces are relatively small compared to $F_{PASSIVE}$, act against each other, and are difficult to estimate.

## Determination of Critical Failure Surface

In order to determine $F_{ULT}$ we consider two possible failure cases, as shown in Figure A3. The case that results in smaller foundation loads is selected for calculation of $F_{ULT}$.  In Case A, a log-spiral based passive pressure is applied to the face of the pile cap.  This passive pressure is combined with the lateral resistance provided by the portion of pile length that extends through the crust.  A side force on the pile cap is added to the passive resistance.

## ATTACHMENT 1

Case B assumes that the pile cap, soil crust beneath the pile cap, and piles within the crust act as a composite block. This block is loaded by a Rankine passive pressure and side force developed over the full height of the block. Rankine passive pressure is assumed in this case because the weak liquefied layer directly beneath the composite block cannot transfer the stresses required to develop the deeper log-spiral failure surface that is generated by wall face friction.

For most practical problems, Case B will result in smaller foundation loads, though the controlling mechanism is dependent on the size and number of piles, and the thickness of crust. The most accurate way to determine the controlling mechanism is to use a pile lateral load analysis program to model each case. For design efficiency, however, an approximate calculation of $F_{ULT}$ for each case can be performed to determine the controlling design case. In most instances, one design case will clearly dominate (typically Case B). If $F_{ULT-A}$ $\approx F_{ULT-B}$ then a more complete comparison can be made by modeling both cases in a pile lateral load analysis program.

The estimation of $F_{ULT}$ for Case A and Case B can be performed as follows:

$$F_{ULT-A} \approx F_{PASSIVE-A} + F_{PILES-A} + F_{SIDES-A} \tag{2a}$$

where $F_{PASSIVE-A}$ is given by equation (3), using $K_p$ (log-spiral).

$$F_{PILES-A} \approx n \cdot GRF \cdot P_{ULT} \cdot L_c \tag{2b}$$

where $n$ is the number of piles, $GRF$ is the group reduction factor defined in Section A-2, $P_{ULT}$ is the ultimate pile resistance determined in Figure A4, and $L_c$ is the length of pile extending through the crust. $F_{SIDES-A}$ is given in equation (8a) or (8b).

$$F_{ULT-B} \approx F_{PASSIVE-B} + F_{SIDES-B} \tag{2c}$$

where $F_{PASSIVE-B}$ is given by equation (3), using $K_p$ (Rankine). $F_{SIDES-B}$ is given in equation (8a) or (8b) but with cap thickness $T$ replaced by the thickness of the composite block ($Z_c − D$ in Figure A2).

**LRFD**
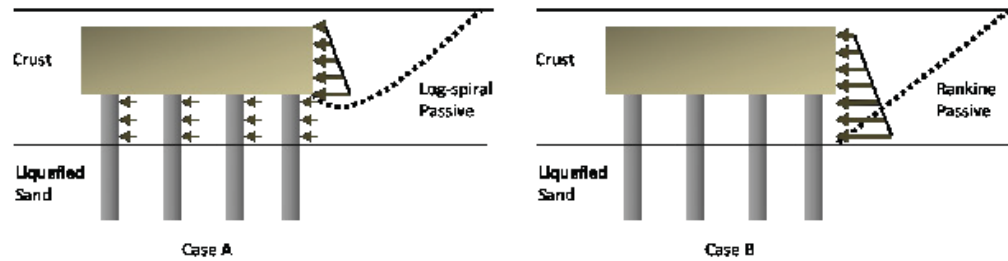
**Figure A3  Two possible design cases for the calculation of
the ultimate passive load due to the soil crust**

Case A considers the combined loading of a log-spiral passive wedge acting on the pile cap and the ultimate resistance provided by the portion of individual pile length above the liquefied zone.

Case B considers the loading of a Rankine passive wedge acting on a composite soil block above the liquefaction zone.

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Estimation of $P_{ULT}$

For SAND:

$P_{ULT} = (C_1 H + C_2 B)\gamma H$

For CLAY:

$P_{ULT} = 9cB$

Where:  $P_{ULT}$ = ultimate lateral resisting force per unit length of pile
$H$  = average pile depth in the crust
$B$  = pile diameter
$\gamma$  = effective unit weight of the crust
$C_1$ = 3.42-0.295 $\phi$ +0.00819 $\phi^2$
$C_2$ = 0.99-0.0294 $\phi$ +0.00289 $\phi^2$
$\phi$  = friction angle of crust          $20 < \phi < 40$
$c$  = undrained shear strength

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**Figure A4  Calculation of $P_{ULT}$ for Sands and Clays, approximated from
formulations by API (1993)**

**$F_{PASSIVE}$ ($c$-$\phi$ soils)**

For soils with a frictional component, $F_{PASSIVE}$ can be estimated using equation (3).

$$F_{PASSIVE} = \left( \overline{\sigma'_v} K_p + 2c' \sqrt{K_p} \right) (T)(W_T)(k_w) \tag{3}$$

In equation (3) $\overline{\sigma'_v}$ is the mean vertical effective stress along the pile cap face, $K_p$ is the passive pressure coefficient, $c'$ is the cohesion intercept, and $k_w$ is an adjustment factor for a wedge shaped failure surface. In general, for cohesionless soil $K_p$ should be based on a log-spiral failure surface. A convenient approximation for $K_p$ (log-spiral) is given in equation (4), where $\phi$ is the peak friction angle of the crust, and $\delta$ is the pile cap-soil interface friction angle (recommended as $\phi/3$ for cases of liquefaction).

If $\phi > 0$:

$$K_p \,(log - spiral) = \mathrm{Tan}^2\left( 45 + \frac{\phi}{2} \right)\left( 1 + \left( 0.8152 - 0.0545\phi + 0.001771\phi^2 \right)\frac{\delta}{\phi} - 0.15(\frac{\delta}{\phi})^2 \right)$$

(valid for $\phi$ ranging from 20° to 45° and $\delta \leq \phi$)

If $\phi = 0$:

$$K_p \,(log - spiral) = 1 \tag{4}$$

For cases where the pile cap or composite cap-pile-soil block (case B) extends to the top of the liquefiable layer, $K_p$ should be calculated using Rankine's formulation, equation (5), instead of a log-spiral solution since the presence of the liquefiable layer impedes the development of the deeper log-spiral failure surface.

$$K_p (\text{Rankine}) = \mathrm{Tan}^2(45 + \frac{\varphi}{2}) \tag{5}$$

A solution for $k_w$, developed by Ovesen (1964), is given in equation (6).

$$k_w = 1 + (K_p - K_a)^{\frac{2}{3}}\left\{ 1.1\left( 1 - \frac{T}{D+T} \right)^4 + \frac{1.6}{1+\frac{5W}{T}} + \frac{0.4(K_p - K_a)\left( 1 - \frac{T}{D+T} \right)^3}{1+\frac{0.05W}{T}} \right\} \tag{6}$$

**$F_{PASSIVE}$ ($c$-only soils)**

For cases where the crust is entirely cohesive (no frictional strength component) $F_{PASSIVE}$ should be estimated using equation (7) (Mokwa et al., 2000)

$$F_{PASSIVE} = \left( 4 + \frac{\gamma(D+T)}{c} + \frac{D+T}{4W_T} + 2\alpha \right) cW_T \frac{(D+T)}{2} \tag{7}$$

In equation (7) $\alpha$ is an adhesion factor and can be assumed to be 0.5.

**$F_{SIDES}$**

$F_{SIDES}$ can be calculated using equation (8a) for effective stress conditions and equation (8b) for total stress conditions. In both instances, $\alpha$ is an adhesion factor assumed to be 0.5. All other variables are defined as in equations (3) and (4).

$$F_{SIDES} = 2(\overline{\sigma'_v}\, \mathrm{Tan}(\delta) + \alpha c')W_L T \tag{8a}$$

$$F_{SIDES} = 2\,\alpha c W_{LT} \tag{8b}$$

## Determination of $\Delta_{MAX}$

Traditionally, passive resistance against a rigid wall will take 1 to 5% of the wall height to fully mobilize. Empirical observation and theoretical studies by Brandenberg (2007) suggest that for the case of a crust overlying a liquefied layer, mobilization of the full passive force may require relative displacements much larger than 5% of wall height. This larger deformation stems from the greatly reduced capacity of the underlying liquefied soil to transmit shear stress from the bottom of the crust. These stresses are thus constrained to spread horizontally (instead of downward) and spread large distances through the crust. This effect is most pronounced when the crust thickness is equal to or smaller than the pile cap thickness and the pile cap width is large relative to the crust thickness. The effect diminishes as the crust becomes thicker relative to both the pile cap thickness and width. This behavior is accounted for in equation (9a) by using the adjustment factors $f_{depth}$ and $f_{width}$. These factors are given in equation (9b) and (9c) and shown graphically in Figure A5. Refer to Figure A2 for parameter definitions used in equations (9a) - (9c).

$$\Delta_{MAX} = (T)(0.05 + 0.45 f_{depth} f_{width}) \tag{9a}$$

$$f_{depth} = e^{-3\left(\frac{Z_c - D}{T} - 1\right)} \tag{9b}$$

$$f_{width} = \frac{1}{\left(\dfrac{10}{\dfrac{W_T}{T}+4}\right)^4 + 1} \tag{9c}$$
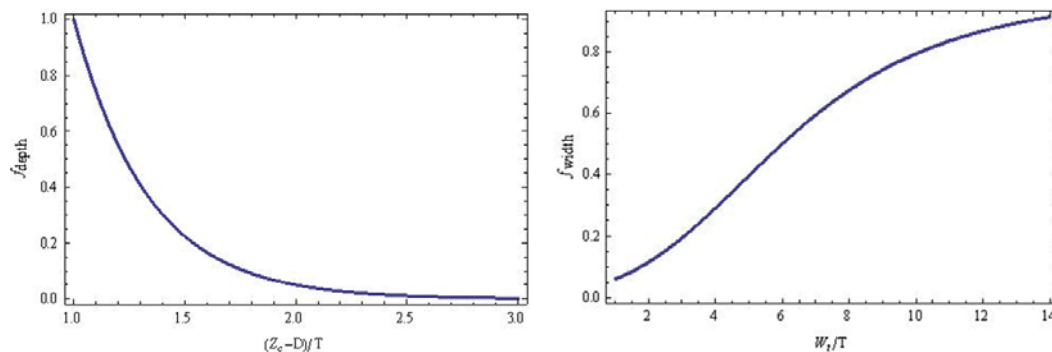
**Figure A5** $f_{depth}$ **as a function of the ratio of crust thickness to pile cap thickness (left)** $f_{width}$ **as a function of the ratio of pile cap width to pile cap thickness (right). (Brandenberg, personal communication)**

# Calculation of $p$ - $y$ Curves for Piles

$P$-y curves are typically generated using a pile lateral load analysis program. The $p$-$y$ models implemented in a pile lateral load analysis program are based on Matlock (1970) (soft clay), Reese et al. (1975) (stiff clay), and Reese et al. (1974) (Sand). If a pile lateral load analysis program is used to perform a single bent analysis, the properties of the bent foundation must be captured by an equivalent (single) superpile. If a global model is used, the analyst has the choice to model each foundation using one or more superpiles, or they can model each pile individually. The corresponding $p$-$y$ curves must be modified to account for the modeling choice. Generally, the "$p$" in the $p$-$y$ curves for a single group pile must be scaled by a factor equal to the number of group piles multiplied by an adjustment factor for group efficiency, or Group Reduction Factor (*GRF*), as given in equation (10).

$$p_{super} = p_{single} \cdot n \cdot GRF \tag{10}$$

## Group Reduction Factors

Piles in groups tend to be less efficient in resisting lateral load, on a per pile basis, than isolated piles. This reduced efficiency results from the overlapping stress fields of closely spaced piles. Leading row piles tend to attract more load than trailing rows, for example, which tend to be shielded by the rows in front of them. In order to match group behavior with a single pile, a composite group efficiency factor, or Group Reduction Factor (*GRF*), must be applied to the individual $p$-$y$ curve as a $p$-multiplier. Caltrans practice is to use $p$-multipliers as a function of pile spacing and transverse oriented row. The $p$-multipliers are obtained from AASHTO Table 10.7.2.4-1 with California Amendments. In order to determine the *GRF*, the factor for each row should be averaged. For example, a 5 row pile group with 3 diameter spacing would have a *GRF* = (0.75 + 0.55 + 0.40 + 0.40 + 0.40)/5 = 0.50.

### p-y Curves for Liquefied Soil

### P-Multiplier ($m_p$) Method

The dramatic strength loss associated with liquefaction can be accounted for through application of p-multipliers that scale down *p-y* curves reflective of the nonliquefied case. Figure A6 shows the range of back-calculated *p*-multipliers (Ashford et al. 2011) from a number of studies. A recommended equation for mp is also given and plotted against the back calculated values. In the equation, *N* refers to the clean sand equivalent corrected blow count $(N_1)_{60CS}$. A clean sand correction by Idriss and Boulanger (2008) is provided in Section A-4. The recommended p-multiplier equation in Figure A6 is appropriate for soils that reach 100% excess pore pressure ratio. In soils that are not expected to fully liquefy but will reach a pore pressure ratio significantly greater than zero, $m_p$ can be scaled proportionately by $100/r_u$ where $r_u$ is the excess pore pressure ratio (percent).
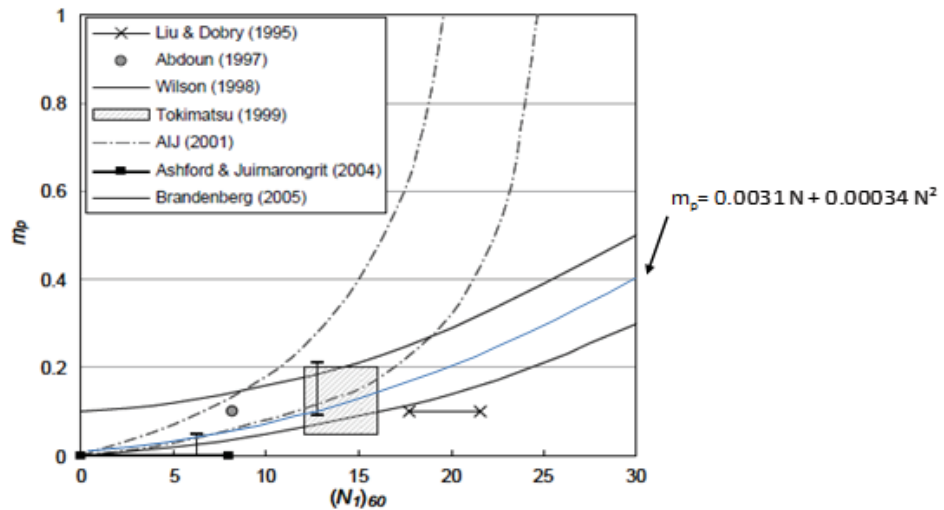


**Figure A6  p-multiplier ($m_p$) vs. clean sand equivalent corrected blow count, $(N_1)_{60CS}$, from a variety of studies. An equation is given for the recommended design curve**

### Residual Strength Method

An alternative to the *p-multiplier* method is to develop *p-y* curves based on soft clay *p-y* models (e.g. Matlock 1970) where the residual strength of the liquefied soil is used in place of the undrained shear strength of the soft clay. Residual strength can be estimated using the following relation by Kramer and Wang (2015):

$$S_r = 2116 \cdot \exp\left(-8.444 + 0.109(N_1)_{60} + 5.379\left(\frac{\sigma_v'}{2116}\right)^{0.1}\right) \tag{11}$$

In equation (11) both $S_r$ and $\sigma_v'$ are in units of psf. The SPT blow count in this relation does not require adjustment for fines content. It is recommended that $\varepsilon_{50} = 0.05$ be used when applying the Matlock soft clay procedure.

## Modification to *p-y* Curves Near Liquefaction Boundary

The occurrence of liquefaction will affect the potential lateral resistance of nonliquefied layers directly above or below the liquefied strata. *p-multipliers* can be used to reduce the subgrade reaction of nonliquefied soils in the vicinity of a liquefied layer as shown in Figure A7.
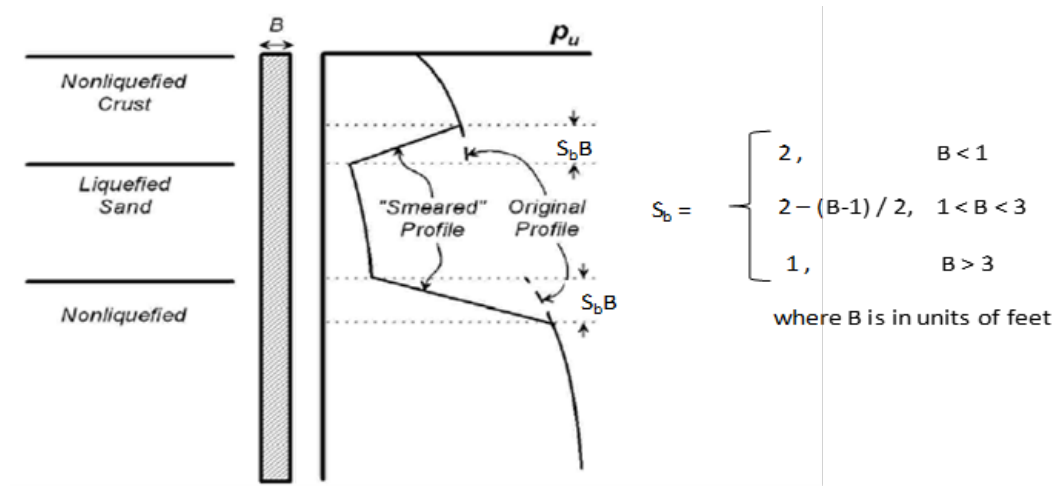


**Figure A7  Modification to the ultimate subgrade reaction, $p_u$, to account for the weakening effect the liquefied sand exerts on overlying and underlying nonliquefied strata**
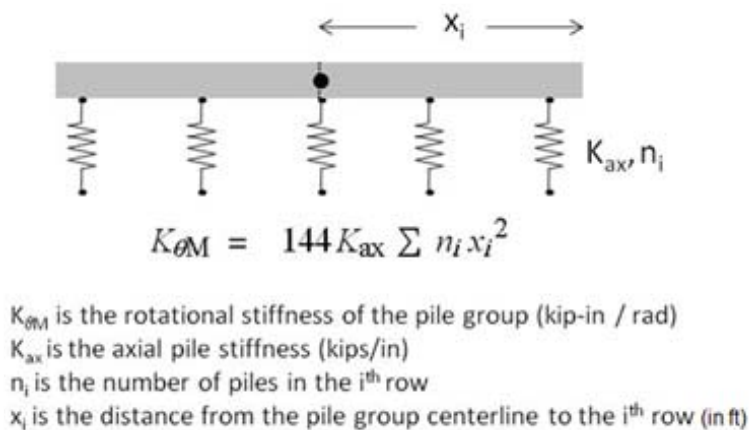
If $z$ is the distance (in feet) above or below the liquefaction boundary and $p_{u-L}$ and $p_{u-NL}$ are the ultimate subgrade reactions in the adjoining liquefied and nonliquefied layers, respectively, then a $p$-multiplier ($m_p$) should be applied as given in equation (12). This $p$-multiplier should be applied at increasing distance from the liquefaction boundary until it equals 1.

$$m_p = \frac{p_{u-L}}{p_{u-NL}} + \left(1 - \frac{p_{u-L}}{p_{u-NL}}\right)\left(\frac{z}{s_b B}\right) \tag{12}$$

# Determination of Rotational Stiffness $K_\theta$

Estimation of the rotational stiffness of a pile group can be simplified by assuming that the axial stiffness of a pile is the same in uplift and compression. If this assumption is true, or approximately true, the foundation will rotate about its center and the rotational stiffness can be estimated as shown in Figure A8. If the axial stiffness of the pile is considerably larger in compression (due to large end bearing) then the rotational stiffness of the pile group is best estimated using a pile group analysis program (e.g. GROUP). $K_{ax}$ can be estimated by assuming that 75% of the ultimate pile capacity is achieved at 0.25-inch axial displacement. For the case of a Class 200 pile, this corresponds to $K_{ax}$ = 0.75 (400 kips)/0.25 in = 1200 kips/in.



$$K_{\theta M} = 144 K_{ax} \sum n_i x_i^2$$

$K_{\theta M}$ is the rotational stiffness of the pile group (kip-in / rad)
$K_{ax}$ is the axial pile stiffness (kips/in)
$n_i$ is the number of piles in the $i^{th}$ row
$x_i$ is the distance from the pile group centerline to the $i^{th}$ row (in ft)

**Figure A8  Calculation of rotational stiffness of the pile group. The method assumes that the single pile compressive stiffness is approximately equal to the uplift stiffness**
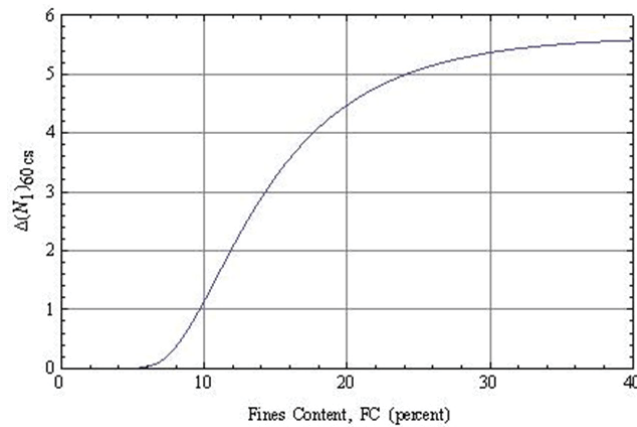
# Idriss and Boulanger (2008) Clean Sand Fines Correction

$$(N_1)_{60\,CS} = (N_1)_{60} + \Delta(N_1)_{60} \tag{13}$$

$$\Delta(N_1)_{60} = \mathrm{Exp}\left(1.63 + \frac{9.7}{FC + 0.01} - \left(\frac{15.7}{FC + 0.01}\right)^2\right) \tag{14}$$

In equation (14), *FC* is the percent fines smaller than the #200 sieve. This relation is plotted in Figure A9.



**Figure A9  Variation of $\Delta\,(N_1)_{60}$ with fines content (*FC*)**
**(from Idriss and Boulanger, 2008)**

# References

Caltrans, (2014). *California Amendments to AASHTO LRFD Bridge Design Specifications*, 6th Edition. California Department of Transportation, Sacramento, CA.

Ashford et al., (2011). *Recommended Design Practice for Pile Foundations in Laterally Spreading Ground*, Pacific Earthquake Engineering Research Center, PEER Report 2011/04.

Brandenberg, SJ, et al, (2007). *Liquefaction-Induced Softening of Load Transfer between Pile Groups and Lateral Spreading Crusts*, Journal of Geotechnical and Geoenvironmental Engineering 133 (1) pg. 91-103.

Idriss, I.M., and R.W. Boulanger, (2008). *Soil Liquefaction During Earthquakes*, EERI Monograph #12, 499 14 th Street, Suite 320, Oakland, CA.

Kramer, S. and Wang, C.H., (2015). *Empirical Model for Estimation of the Residual Strength of Liquefied Soil*, Journal of Geotechnical and Geoenvironmental Engineering.

Matlock H., (1970). *Correlations of Design of Laterally loaded Piles in Soft Clay*, Proc. Offshore Tech. Conference, Houston Texas, 1(1204):577-594.

Mokwa, R.L, and J.M. Duncan, (2000). *Investigation of the Resistance of the Pile Caps and Integral Abutments to Lateral Loading*, FHWA Report No. VTRC 00-CR4, FHWA 400 North 8th Street, Room 750, Richmond, VA.

Reese, L. C. and Welch, R. C., (1975). *Lateral Loading of Deep Foundations in Stiff Clay*, Journal of Geotechnical Engineering Division, ASCE, Vol. 101, GT7, pp. 633-649.

Reese, L. C., Cox, W. R., and Koop, F. D., (1974). *Analysis of Laterally Loaded Piles in Sand*, Proc. 6th Offshore Technology Conference, Paper 2080, Houston, Texas, 473-483.